GUIDANCE FOR METACOGNITIVE JUDGMENTS: A THINKING-ALOUD ANALYSIS IN MATH PROBLEM SOLVING

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Abstract: The aim of the present study was to examine the effect of an intervention program –based on the IMPROVE guidance for metacognitive judgments– on fourth-grade students (n = 13), compared to a control group (n = 13) that was not exposed to metacognitive guidance. A qualitative analysis was performed on the thinking-aloud process data in the three phases of Self-Regulated Learning (SRL) –planning, monitoring and reflection– while solving a non-routine math problem. The process analysis in each phase focused on SRL statements referring to metacognition (knowledge of cognition and control), motivation (mastery goals and self-efficacy), and metacognitive judgments, namely, ease of learning, judgment of learning, feeling of knowing, and confidence. The findings indicated that students in the intervention program achieved more correct solutions on the non-routine problem than the control group. Moreover, there were differences regarding statements related to SRL and metacognitive judgments in the different SRL phases. The implications of the study are discussed.

Key words: Math problem solving, Metacognitive guidance, Qualitative analysis, Self- judgment, SRL phases, Think aloud

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INTRODUCTION

Self-regulated learning (SRL) has gained theoretical and research prominence in mathematics learning-related research in the last 20 years. Researchers (e.g., Schoenfeld, 1992) argued that students' difficulties in mathematics are not always a result of lack of knowledge, but rather of the ineffective *activation* of their knowledge due to students' inability to monitor and control their solution processes. As such, it is suggested that students need to learn how to self-regulate their learning to improve mathematical problem solving and SRL.

SRL is a cyclical process, which includes metacognition, metacognitive judgments and motivation, besides other components, that is not achieved spontaneously by students (Pintrich, Wolters, & Baxter, 2000; Zimmerman, 2008). Metacognition is conceived as a major component in SRL process (National Council of Teachers of Mathematics (NCTM), 2000; Program for International Student Assessment (PISA), 2003). However, while guidance oriented to metacognition and motivation is largely investigated in relation to mathematics problem solving (Dignath, Büttner, & Langfeldt, 2008; Kistner et al., 2010; Mevarech & Kramarski, 2014), only few studies examined the role of metacognitive judgments in mathematical problem solving, particularly among young students (age 9 years). The existing studies mainly examined metacognitive judgments such as JOL (Judgment of Learning) and CJ (Confidence Judgments), largely assessed by self-report measures (Fernández, Kroesbergen, Pérez, González-Castro, & González-Pienda, 2015; Jacobse & Harskamp, 2012; Labuhn, Zimmerman, & Hasselhorn, 2010; Roebers, Krebs, & Roderer, 2014) while neglecting judgments such as EOL (Ease of Learning) and FOK (Feeling of Knowing).

The present study aimed to investigate a unique program for "Metacognitive Judgment Guidance" based on the IMPROVE metacognitive model (Kramarski & Mevarech, 2003) oriented to the three SRL phases and to examine, in a thinking aloud qualitative analysis, its effect on a novel problem-solving task via the SRL components: Metacognition, metacognitive judgments (four types) and motivation, compared to a control group not exposed to metacognitive scaffold.

In what follows we shall, firstly, present an overview of the SRL theory and its components; secondly, measures of SRL processes; thirdly, the "IMPROVE Guidance for Metacognitive Judgments", and, finally, the aims of the present study, its assumptions, and findings.

Self-regulated learning

There is broad consensus among researchers that SRL is associated with academic achievement in different domains, so that learners with high SRL ability are characterized by high academic achievements (Azevedo & Cromley, 2004; Kramarski & Gutman, 2006; Winne & Hadwin, 1998; Zimmerman, 2000). Although there is no single conceptualization of SRL, it seems that there are two main perspectives – one focusing on the SRL process (Zimmerman, 2000) and the other on its components (Pintrich, 2000).

Consistent with Zimmerman's theory (2008), SRL involves proactive, constructive processes where the learner *sets goals* and attempts to *monitor* and *evaluate* one's own cognition, metacognition and motivation, subject to contextual features. SRL follows a cyclical process that includes three phases: (see Figure 1): forethought (planning), performance (monitoring) and evaluation (reflection). These phases are manifested in the learner's planning of work by using strategies, monitoring, and reflecting on the process of learning via judgments on what was done and what could be modified. On the other hand, theoreticians focusing on SRL components emphasize the mutual and continuous interaction between personal, behavioral and environmental factors (Bandura, 1986) and pinpoint to the three main components of SRL, namely, cognition, metacognition and motivation (Pintrich, 2000).

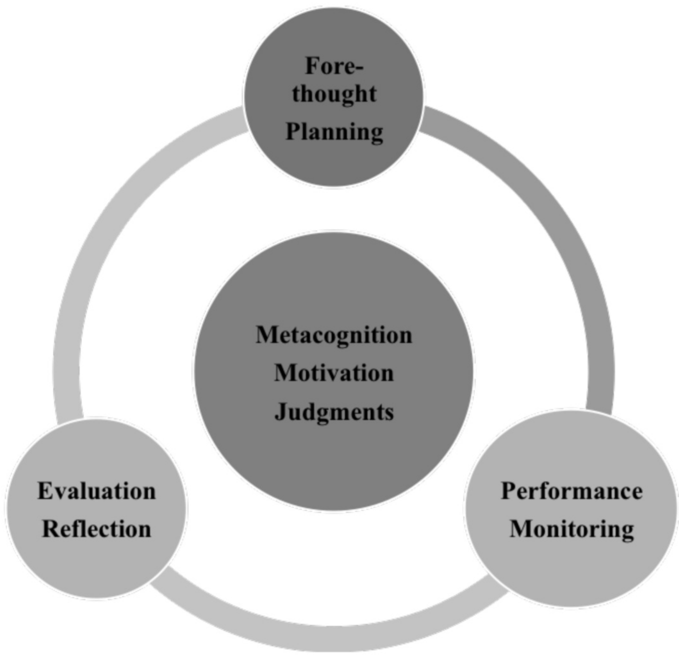

Figure 1. The cyclical model of Self-Regulated Learning (Zimmerman, 2000).

Metacognition

Metacognition is defined as cognition about cognition (Flavell, 1979). According to Efklides (2008), metacognition is a "representation of cognition, and metacognition and cognition are connected through the monitoring and control functions" (Efklides, 2008, p. 278). Flavell and his colleagues (Flavell, Miller, & Miller, 2002) distinguished two major components of metacognition: metacognitive *knowledge* and metacognitive *monitoring* and *self-regulation*, conceived as metacognitive *skills*. A third aspect of metacognition is *metacognitive experiences* (Efklides, 2008; Flavell et al., 2002) or *self-judgments* (Pintrich et al., 2000). Since this study focused on learners' self-judgments, it follows the structure proposed by Pintrich and colleagues (2000) referring to the three components in metacognition: metacognitive *knowledge*, metacognitive *judgments and monitoring*, and metacognitive *control.*

Metacognitive knowledge

Metacognitive knowledge is knowledge of the cognitive characteristics of the person, the task and the strategies used in cognitive tasks (Zohar, 2011). Knowledge of cognition comprises of declarative knowledge about strategy/task ("what"?), procedural knowledge used on various cognitive strategies – ("how"?), and conditional knowledge ("when"? and "why"?) that is important for the flexible and adaptive use of various cognitive strategies (Pintrich, 2002; Schraw, 1998; Zohar & Ben David, 2008).

Metacognitive judgments and monitoring

Compared to metacognitive knowledge that has static nature, metacognitive judgments are more process-related and reflect metacognitive awareness/ experiences in ongoing metacognitive activities that individuals may engage in as they monitor and perform a task (Pintrich et al., 2000).

Metacognitive monitoring evaluates the student's understanding of the task processing and planning, performance and reflection (Destan, Hembacher, Ghetti, & Roebers, 2014; Dunlosky & Metcalfe, 2009; Pintrich et al., 2000; Sobocinski, Malmberg, & Järvelä, 2017; Zimmerman, 2008). Monitoring has been examined prospectively or retrospectively while referencing before, during, or after completing a task, via metacognitive judgments (Mihalca, Mengelkamp, & Schnotz, 2017). There are four types of metacognitive judgments of experiences or processes (Pintrich et al., 2000): Ease of Learning (EOL), Judgment of Learning (JOL), Feeling of Knowing (FK) and Confidence Judgments (CJ).

Ease of Learning (EOL) refers to **prospective** evaluation of the ease of processing of the task at hand (Burkett & Azevedo, 2012; Pintrich et al., 2000). This judgment is made **before** actual processing of the learning task begins ("*Is the task hard or easy for me?*"; "*What will be easy/difficult to learn*"?) (Burkett & Azevedo, 2012; Jemstedt, Kubik, & Jönsso, 2017; Mihalca et al., 2017; Pintrich et al., 2000). Studies conducted among college students indicated that this judgment is subject to improvement after explicit lengthy practice integrated with content teaching (e.g., Cao & Nietfeld, 2005).

Judgment of Learning (JOL) refers to the learner's awareness of the learning strategy they chose and the **prospective** judgment of the efficiency of the work path selected. For instance, if a student tells him/herself: "*I do not understand this problem*", it may lead him/her to re-read the problem again (Azevedo, Cromley, Moos, Greene, & Winters, 2011). JOL also refers to the learner's ability to estimate the future memorability of a learning material or recognize studied information (Burkett & Azevedo, 2012; Jemstedt et al., 2017; Mihalca et al., 2017).

JOLs usually occur in the learning process during the **performance** phase (Jemstedt et al., 2017; Pintrich et al., 2000). Students with overestimated JOLs are likely to abandon their task processing prior to gaining precise control (Isaacson & Fujita, 2006). For this reason, researchers recommend cultivating this type of judgment among undergraduate students. Studies examining interventions for the improvement of JOL accuracy indicated that they are not always successful in achieving their goals (Logan, Castel, Haber, & Viehman, 2012; Townsend & Heit, 2011).

Feeling of Knowing (FOK) is a **prospective** judgment, which "occurs when a person cannot recall something when called upon to do so, but know he knows it, or at least has a strong feeling that he knows it" (Pintrich et al., 2000, p. 49). FOK is manifested *during* the performance phase when learners ask themselves: "*Have I read, heard, or inspected something in the past about this problem*?" (Azevedo et al., 2013). Prior research indicated difficulty in cultivating this type of judgments. For instance, Hicks and Marsh (2002) examined FOK judgments among university students and found no significant correlation between these metacognitive judgments and content acquisition.

Confidence judgments (CJ) is a **retrospective** evaluation, which refers to the confidence of the learner in their response and is mostly realized at the *end* of the learning process *("I'm sure I was right")* (Mihalca et al., 2017; Morgan, Kornell, Kornblum, & Terrace, 2014). This type of judgment estimates the correctness of a response just given (Roebers et al., 2014). Studies indicated a high tendency of overestimation in CJ among students. A learner with a high CJ is likely not to return to seek and correct their mistakes (Shin, Bjorklund, & Beck, 2007). Previous research indicated that the CJs can be developed among children of younger ages, ranging between 9 and 10 years (e.g., Roebers et al., 2014).

Metacognitive control

Similarly to metacognitive judgments and monitoring, metacognitive control is "a process, an ongoing activity" (Pintrich et al., 2000, p. 50). Metacognitive control refers to the selection and use of learning and thinking strategies to repair or correct a mistake. Control is associated with metacognitive monitoring, and judgments that inform of cognitive processing and performance (Nelson & Narens, 1994).

Motivation

Motivation in a learning context refers to the effort of the learner to achieve goals they perceive as meaningful and valuable (Johnson & Johnson, 2003). There are various motivational theories tapping motivational phenomena in learning. One of the most researched theories in education is the achievement goals theory, on which Pintrich and colleagues based their model (Pintrich et al., 2000) for SRL. Another highly important person characteristic that impacts learning behavior is self-efficacy (Bandura, 1986). These theoretical frameworks guided the present study.

Achievement goals theory (Elliot, 1997; Urdan, 1997) focuses on the reasons students have for engaging or not in a learning situation (see also Kaplan & Maehr, 2002). Initially, two main categories of achievement goals were distinguished: mastery (also known as learning goals) and performance (Dweck, 1986). Later on, performance goals were distinguished as performance-approach and performance-avoidance goals (the three-partite model, Midgley et al., 2000). Finally, the approach-avoidance tendencies were used to differentiate the mastery goals as well (Elliot, 1997; Pintrich, 2000). The present study adopted the basic distinction as suggested by Dweck (1986), namely, mastery and performance goals, since in qualitative data it is often difficult to distinguish in fine detail the goals adopted by students. Mastery goals assist the learners to develop their potential, improve their performance, advance their learning, and achieve a deeper understanding. Learners with mastery goals invest effort to better comprehend the learning material, persist in face of difficulties, and use learning strategies for organizing their learning and connecting it to prior knowledge. Learners driven by performance (approach) goals aim to outperform others, exhibit their ability, and achieve high grades. In contrast, students with performance (avoidance) goals aim to avoid exhibiting lack of ability, invest minimal effort and use surface strategies.

Self-efficacy, on the other hand, depicts the person's sense of competence to bring about a specific outcome (Bandura, 1986). Self-efficacy is related to self-concept and mediates the effect of competence on performance (Tzohar-Rozen & Kramarski, 2013). The learner's self-efficacy refers to one's beliefs that they can carry out learning tasks and achieve specific learning outcomes (Bandura, 1977).

Studies on SRL indicated that learners often have difficulty to attain SRL components related to metacognition (knowledge, judgments, monitoring and control), particularly in regard with judgment accuracy (Isaacson & Fujita, 2006; Jemstedt et al., 2017; Mihalca et al., 2017). Other difficulties include engaging in mastery goals to achieve a deeper understanding and carrying out demanding programs for cultivating SRL. Existing intervention programs examined parts of the SRL process (Fernández et al., 2015; Jacobse & Harskamp, 2012; Labuhn et al., 2010; Mevarech & Kramarski, 2014), including self-judgments. However, the empirical evidence regarding cultivation of the different types of metacognitive judgments is inconsistent (Roebers et al., 2014; Roderer & Roebers, 2010). Furthermore, studies that target all SRL components among young students in mathematics with authentic SRL process measurements, is lacking.

Measurement of SRL processes

One of the major issues in SRL research is the measurement of SRL processes that can represent in greater detail the mechanisms behind success or lack of it during learning and self-regulation (Zimmerman, 2008). Often, a procedural approach that emphasizes processes during the phases of SRL, that is, forethought (planning), performance (monitoring) and reflection (evaluation), is adopted (Azevedo, 2014; Pintrich, 2000; Zimmerman, 2000).

Authentic measures in real time are highly valued in SRL research and form valid tools for comprehending learning and considering the complexity of self-regulation (Azevedo, 2014; Greene & Azevedo, 2010). An example of such a tool is "thinking aloud" protocols (Veenman, Wilhelm, & Beishuizen, 2004). The thinking aloud protocols offer spontaneous, non-prompted reporting while the learner report their problem solving in real time. The thinking aloud method can reveal, in real time, the learner's goal setting for the task, comprehension of the problem, planning of the solution, choice of strategy, monitoring of learning, related metacognitive judgments and, finally, control decisions. Moreover, it can bring to the fore the student's ways of dealing with difficulties and motivation (Greene & Azevedo, 2007). Depth pattern analysis for the thinking-aloud method contributes additional perspectives to understanding the dynamic processes of SRL phases. The present study adopted the thinking aloud methodology for evaluating SRL processes.

For the purposes of the present study, a mathematical non-routine problem was chosen, requiring the learner to apply logical considerations, systematic thinking, transfer of knowledge, skills for new situations as well as ability to combine skills. To solve non-routine problems, high-level cognitive skills, such as application of mathematical procedures in an unfamiliar or complex context, are required (Mullis, Martin, & Foy, 2008). In these types of problems, the learner is required to be *alert* during all three SRL phases during the entire learning process and to make problem-solving decisions in which metacognitive judgments are involved. However, these SRL processes are not spontaneously developed.

Previous intervention programs in the mathematics domain focused on the development of metacognitive skills (e.g., monitoring, control) that led to achievement improvement (e.g., Dignath et al., 2008; Kistner et al., 2010; Kramarski, Weisse, & Sharon 2013; Kramarski & Fridman, 2014; Kramarski & Mevarech, 2003; Mevarech & Kramarski, 2014). However, the interventions did not aim at cultivating explicit metacognitive judgments. The multitude of judgment types and the difficulty to precisely differentiate between them make scaffolding difficult in the SRL process, particularly among younger students (Huff & Nietfeld, 2009; Kasperski & Kathir, 2013; Linden & Roebers, 2006; Metcalfe & Finn, 2012).

Based on the aforementioned, the present study aimed to apply a metacognitive program based on the **IMPROVE** model (Kramarski & Mevarech, 2003; Mevarech & Kramarski, 1997, 2014) with an additional focus on guidance for metacognitive judgments.

IMPROVE guidance for metacognitive judgments

The IMPROVE model was designed (Mevarech & Kramarski, 1997, 2014) to foster learners' SRL by scaffolding key aspects of metacognition oriented to problem solving during the three phases of *planning, monitoring,* and *reflection*. The model uses generic self-directed *What, When, Why* and *How* question prompts: *Comprehension* metacognitive questions help learners understand necessary information (e.g., "**What** is the task/problem?"; "Do I understand?"). *Connection* questions help understand the task's deeper level relational structures (e.g., "**What** is the difference/similarity?", "**How** do I justify my conclusion?"). *Strategy* questions help *plan* and select appropriate strategies (e.g., "**What** is the strategy?", "Why?"). *Reflection* questions help *monitoring and controlling* effectiveness (e.g., "Does the solution make sense?", "Can the task be solved otherwise?", **How**?).

The "IMPROVE guidance for metacognitive judgments" was conducted in two steps. First, the IMPROVE model was presented and practiced to create a metacognitive awareness and understanding regarding the three SRL phases, self-questions and their contribution to problem solving. Second, the four metacognitive self-judgments were presented, explained and practiced with "Judgmental rulers" oriented to the SRL phases while integrating the four types of metacognitive judgments

(EOL, JOL, FOK, CJ). Details on the full program are presented in the Method section and Appendix.

The aim of the present study

The present study aimed to examine, in a thinking aloud process, the effects of the "IMPROVE Guidance for Metacognitive Judgments" intervention, oriented to SRL phases, compared to a control group which was not exposed to metacognitive guidance. Hereafter, the following two parts of the program will be called "Metacognitive Guidance".

The effects of the intervention were examined on the following:

- Solving a non-routine math problem;
- Metacognition (knowledge; monitoring and control);
- Motivation (mastery goals and self-efficacy);
- Metacognitive judgments (EOL, JOL, FOK, CJ).

Based on empirical studies, we expected that the IMPROVE group will achieve higher scores in the non-routine problem solving task, and the thinking aloud protocol will indicate more evidence (statements) relating to metacognition and motivation along the SRL phases, compared to the students in control group (e.g., Kramarski & Mevarech, 2003; Mevarech & Kramarski, 1997, 2014; Tzohar-Rozen & Kramarski, 2014). We have not formulated a hypothesis regarding metacognitive judgments since the findings of previous studies are inconclusive or are lacking in the field of mathematics among young ages (e.g., Roebers et al., 2014).

METHOD

Participants

The participants were 26 students (13 girls and 13 boys) from two fourth-grade classes, who attended a school in Northern Israel. Their age ranged between 9-10 years. The students were randomly divided into two groups (Metacognitive Group and Control Group), each group with 13 students. All students were described by their teachers as good students with similar background in mathematical knowledge, whose grades ranged between 80 and 90 out of 100 in a mathematical test administered prior to the intervention. This test was adapted (by changing the numbers) from the standardized Meitzav examination for the fourth grade, originally developed by the Israeli Ministry of Education (2005, Version A). The test included seven problems: five routine problems and two non-routine problems. No significant differences were found between the two groups on the routine and non-routine problems.

The intervention: “IMPROVE Guidance for Metacognitive Judgments”

The duration of the intervention was equal in both groups and lasted 12 hours of teaching distributed to seven sessions. The intervention was carried out by two teachers (each one taught mathematics in one class). Both teachers had a B.Ed. degree and teaching experience of 15 - 20 years. The teachers were guided by the researcher on the relevant intervention program for their class. The researcher was present during the implementation of the intervention program. At the end of each lesson, the researcher provided feedback to the teacher on the program. The assignments were given to the teachers in a bundled folder for each student.

The intervention program in the metacognitive group

First session

The importance of metacognitive guidance was, firstly, explained. This explanation was based on the theoretical framework of the study and was adapted to the students’ age. In this context, the students were presented with the IMPROVE metacognitive self-questions directed to **comprehension, connection, strategy selection and reflection** (Kramarski & Mevarech, 2003; Mevarech & Kramarski, 1997, 2014) in accordance with the three phases of the cyclical model of SRL, namely, planning, monitoring and reflection (Zimmerman, 2000). The explanations regarding each SRL phase and example questions are presented in Figure 2.

Planning – comprehension of concepts, task demands, finding connections.	**Question:** *Did I understand the problem? What is similar and what is different?*
Monitoring – performance processes	**Question:** *What is the strategy I choose? Why?*
Reflection – an examination of the solution process and the result	**Questions:** *Is the solution reasonable? Is there another way for the solution? How?*

Figure 2. Examples of metacognitive questions for each SRL phase (based on IMPROVE model, Mevarech & Kramarski, 1997, 2014)

Additionally, the students were presented with "Judgmental rulers" of four types of metacognitive judgments: Ease of Learning (EOL), Judgment of Learning (JOL), Feeling of Knowing (FOK), and Confidence Judgments (CJ) (see Appendix). The "Judgmental rulers" were presented in accordance to the SRL phases in problem solving (planning, monitoring, reflection): EOL after reading of problem but *before* the solution, JOL and FOK *during* the solution, and CJ *after* the solution. The students were given an explanation, examples and a demonstration of how to mark the judgments on the ruler.

Sessions 2-6

Sessions 2 to 6 were devoted to word problem solving (four sessions for solving routine problems and one session for solving non-routine problems) from the fourth-grade curriculum (Israeli Ministry of Education, 2006). Additionally, the problems were discussed, while implementing metacognitive self-questions. The structure of these sessions comprised three main stages: (a) Repeating the three types of the metacognitive self-questions that corresponded to the three phases of the cyclic model of SRL. (b) Solving four numerical problems while using metacognitive "Judgmental rulers". Each judgment/ruler was part of the solution of one of the four problems. (c) Discussing the solution strategy and the metacognitive self-questions.

Session 7

The last session was devoted to a summary of the intervention with a focus on the contribution of the "IMPROVE Guidance for Metacognitive Judgments" to enhance the students' mathematical understanding.

The intervention program in the control group

The learning program of the control group also lasted 12 hours of teaching and included seven sessions, similarly to the metacognitive group. The program in the control group focused on features of word problems and strategic practice for solving such problems, in accordance to the recommendations included in their textbooks. The students in the control group solved the same four word problems in each lesson, as the metacognitive group. The time for problem solving was the same in both groups. The control group was not exposed to the metacognitive questions or to a "Judgmental ruler" explicitly. They were, however, informed about the importance of the solution evaluation and were asked to think about their solution with a reflection statement (*"To what degree do you believe you managed to solve the problem?").*

Mathematical task and thinking aloud

Following the administration of the intervention, the students were asked to solve a non-routine word problem in the thinking aloud method (Veenman et al., 2004) that represents the learning process in real time. Solving a word problem in a thinking aloud mode served two main aims: First, to examine the effect of the intervention program on students' performance on word problems; second, to examine the effect of the intervention program on self-regulation processes. One of the researchers from the research team met with each student individually, presented them with the assignment and asked them to verbalize the solution path.

There was no time limit for the solution of the problem. Problem solving lasted ten minutes on average. The students' solutions were recorded and transcribed into written documents, including a reference to nonverbal behaviors, which were observed by the researcher during the thinking-aloud process.

The mathematical non-routine word problem to be solved had not been practiced in the classroom (see Figure 3). The problem-solving process was analyzed with respect to the correctness of the solution, metacognition (i.e., knowledge of cognition: declarative knowledge, procedural knowledge, and control), motivation (i.e., mastery goals and self-efficacy), and judgments (i.e., EOL, JOL, FOK, and CJ).

The teacher arranged the students of the class in groups of 5 so that one student remained alone.

When the teacher rearranged the same children in the same class in groups of 6, all students were part of a group.

How many students are in the classroom? Explain.

Figure 3. The mathematical problem used after the intervention

Scoring

1. *Task performance*: The correct answer was given a score of 1 and the wrong answer 0. The percentage of students who gave a correct response in each group was calculated.
2. *Thinking aloud*: The transcripts of students from both groups were processed in two stages:
   (a) A content analysis of all documents was first conducted (Shkedi, 2005). The unit of analysis was a phrase expressing one idea; most thinking aloud statements consisted of simple or complex sentences. When a complex

sentence included more than one ideas, it was divided into several phrase units.

(b) Categorization of the phrase units. This process also made use of non-verbal behaviors/ student responses informing about the meaning of the phrase with respect to the SRL processes (Pintrich et al., 2000). The following components were categorized: *metacognition* (knowledge of cognition and control), *motivation* (mastery goals and self-efficacy), and metacognitive judgments (see Figure 4).

SRL phases		Planning	Monitoring	Reflection
Components of self-regulation in learning	**Metacognition:** ***knowledge of cognition; control***	**"I will understand better after reading once more"** (control) **"Firstly, you picture the situation"** (procedural knowledge)	**"the data doesn't add up for me"** (control) **"I think I should start with multiples of 6"** (procedural knowledge)	**"I examined all of the options"** (control)
	Motivation: mastery *goals; self-efficacy*	**"I am good with problems"** (self-efficacy)	**"I want to solve this problem"** (mastery goals)	**"After I solve this problem, I think I will solve everything!"** (self-efficacy)
	Metacognitive Judgments: **EOL – *Ease of Learning;*** **JOL – *Judgment of Learning;*** **FOK – *Feeling of Knowing;*** **CJ – *Confidence Judgments***	**"I will succeed well in finding the solution"** (EOL)	**"I am not sure that I will succeed in solving the problem, the solution I had in mind is not right"** (JOL) **"I don't think I've seen problems of this kind before"** (FOK)	**"I know that there are no other solutions"** (CJ)

Figure 4. Examples of thinking-aloud statements indicative of self-regulated-learning components as a function of the three SRL phases

To examine the reliability of the categorization of thinking-aloud statements, inter-rater agreement was used. Cohen's Kappa was computed and indicated high agreement regarding metacognition (91%), motivation (88%) and self-judgments (88%), during planning, monitoring and reflection, respectively.

Fidelity of the intervention

To check the fidelity to the instructions of the program, each week during the intervention (approximately two months), one of the researchers of the team visited one of the participating classes and evaluated the implementation of the program according to the aforementioned criteria.

RESULTS

Mathematical performance

In calculating the frequencies of participant' correct answers, it was found that in the Metacognitive Group there were 70% correct answers compared to only 12% correct answers in the Control Group.

Qualitative analysis of the SRL processes

Henceforth, phrases related to metacognition (knowledge of cognition and control of cognition) are referred to as *metacognitive statements*; phrases related to motivation (mastery goals and self-efficacy) are referred to as *motivational statements*, and phrases related to metacognitive judgments are referred to as *judgmental statements.* First, we will present the findings related to metacognition, followed by motivation and finally judgments.

Metacognitive statements in each of the three SRL phases

Figure 5 indicates that the students in the Metacognitive Group used metacognitive statements in all SRL phases, while the students in the Control Group did not use them in the reflection phase. The figure also indicates that the incidence of the

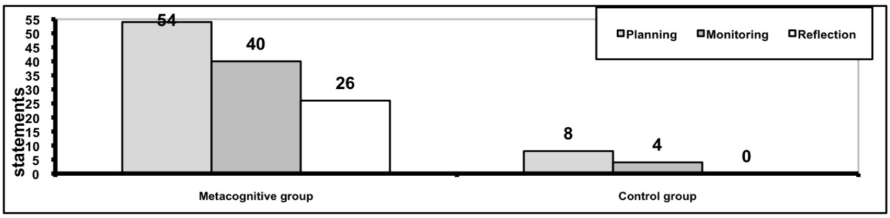

Figure 5. Frequency of metacognitive statements as a function of SRL phase in the Metacognitive and Control group

metacognitive statements in the Metacognitive Group (n = 120) was higher than in the Control Group (n = 12).

Planning phase

In this phase, 62 (in total) metacognitive statements were identified in the *two* groups. Fifty-four of the metacognitive statements (87%) came from the Metacognitive Group and 8 (13%) statements from the Control Group (see Figure 5). All students in the Metacognitive Group used these types of phrases. Specifically, each participant used about five phrases compared to less than half of this number used by the students in the Control Group (where each participant used about one sentence). The distribution of metacognitive statements is presented in Table 1.

Table 1. Frequency and percent (%)* of SRL-related statements as a function of group and phase

SRL phases	Metacognitive Group	Control Group	Total
Metacognitive statements			
Planning			
Frequency (%)	54 (100%)	8 (46%)	62
Mean	4.46	0.61	
Monitoring			
Frequency (%)	40 (100%)	4 (30%)	44
Mean	3.08	0.31	
Reflection			
Frequency (%)	26 (84.6%)	0	26
Mean	2.0	0	
Motivational statements			
Planning			
Frequency (%)	3 (15%)	1 (7.7%)	4
Mean	0.23	0.08	
Monitoring			
Frequency (%)	2 (15%)	0	2
Mean	0.15	0	
Reflection			
Frequency (%)	2 (15%)	0	2
Mean	0.15	0	
Judgmental statements			
Planning			
Frequency (%)	25 (92%)	8 (54%)	33
Mean	1.92	0.61	
Monitoring			
Frequency (%)	23 (92%)	19 (77%)	42
Mean	1.77	1.46	
Reflection			
Frequency (%)	9 (62%)	6 (38%)	15
Mean	0.69		

Note: Mean refers to the mean of phrases per participant

* Percent is calculated of the total amount of students who used these types of sentences

Furthermore, in the Metacognitive Group 58 (93.5%) planning statements referred to *metacognitive knowledge* (declarative knowledge and procedural). However, only four (6.5%) of all planning statements referred to the *control* of cognition.

Examples of metacognitive statements in the planning phase in each of the groups were as follows:

> Tali (Metacognitive Group): *I will read the question, I need to think first in order to understand it well* (declarative knowledge).
>
> Meni (Metacognitive Group): *I understand that I actually need to solve it from the end* (control).
>
> Shelly (Control Group): *Can I not read aloud? I want to read in my head* (procedural knowledge).

The analysis of the metacognitive statements indicated that the students attempted to deepen their understanding of the problem and searched for effective strategies. Specifically, the students in the Metacognitive Group tried to deepen their understanding of the problem and emphasized the importance of understanding in the phase of planning the solution. In addition, in this group, the students implemented and internalized the strategies acquired during the intervention program, such as recreation of the problem space (procedural knowledge). Moreover, they searched for a strategy for the solution while attempting to understand the data of the problem. They also used more accurate mathematical language. At this phase, the Control Group did not demonstrate any reference to effective strategies for the solution or effective learning strategies.

Monitoring phase

In this phase, 44 (total) metacognitive statements were identified in the *two* groups. Forty statements (91%) were found in the Metacognitive Group and four in the Control Group (9%). *All* students in the Metacognitive Group used these types of statements, with a mean of two statements per student, compared to a third of the students in the Control Group (see Table 1). Additionally, the metacognitive statements at this phase referred to *metacognitive knowledge* (declarative and procedural) and to *control* of cognition. Forty-four statements were found in both groups, as different frequency of 30 (68%) belonged to *metacognitive knowledge* and 14 (32%) to *control*. Moreover, the statements found in the Control Group referred to knowledge of cognition only.

Herein are examples of metacognitive statements used in the Metacognitive Group during the monitoring phase:

> Noy (Metacognitive Group): *If I multiple 6 by 5 it's not good because 30 divides precisely and I need one child to remain alone* (control).

> Ariel (Metacognitive Group): *I think I should check for multiples of 6* (procedural knowledge).
>
> Meni (Metacognitive Group): *The answer 6 is correct according to the data* (control) *but it doesn't make sense that a classroom has 6 students* (control).

The above statements indicate that the students in the Metacognitive Group examined the effectivity of their chosen strategy (procedural knowledge) and matched the data of the problem (control). The students used correct mathematical language and an activation of deep level control was visible during the solution. Specifically, the suggested solutions reflected numeric insight, which was evident in the unique path of the solution and the explanation provided.

Herein are examples of metacognitive statements in the monitoring phase in the Control Group:

> Daniel (Control Group): *I think I need to multiply* (procedural knowledge).
>
> Naomi (Control Group): *I will examine the multiple operation* (procedural knowledge).

In the metacognitive statements of the Control Group, the need for choosing a strategy for dealing with the content of the problem was evident; however, there was insufficient verbalization of the choice process and its control. The metacognitive statements did not indicate a deeper analysis of the problem data or attempt to understand it. Only students of the Metacognitive Group demonstrated a numeric insight that was evident in the unique ways of solving the problem while using a rich mathematical language to verbalize the thinking processes.

Reflection phase

Metacognitive statements at this phase were found *only* in the Metacognitive Group. In this category, 26 statements were found (by 11 students, each student used approximately two statements (see Table 1). All of them belonged to the *control* of cognition.

Following are examples of metacognitive statements in the reflection phase:

> Oded (Metacognitive Group): *The answer is 36. It divides by 6 with no remainders and by 5 with a remainder of 1. I checked it and the solution is correct* (control).
>
> Gal (Metacognitive Group): *36 is good because at first one child remains and afterwards everyone divided* (control).

The metacognitive statements in the Reflection phase indicated that the students in the Metacognitive Group had control of the solution and examined it against the verbal and numeric data of the problem. It is further clear that the students could verbalize their thinking process and recapitulated the main steps of their solution. The statements further revealed reflection over the entire process of the solution while confirming its correctness. Evidently, the students in this group internalized and implemented the principles of the intervention program (IMPROVE guidance) in which they had participated.

In conclusion, the metacognitive statements belonging to the three phases of SRL exhibited differences between the Metacognitive and the Control groups in metacognitive knowledge and control of cognition, both in the quantity and quality of conceptualizing statements. These differences can be attributed to the nature of the intervention program implemented in the Metacognitive group as discussed hereinafter.

Motivational statements in each of the three problem solving phases

Figure 6 indicates that only the students in the Metacognitive Group used motivational statements (n = 7 by 5 students) in

Motivational statements were found in the planning phase in both groups. However, their frequency was low: three statements in the Metacognitive Group (by two students) and in the Control Group one (see Table 1). Two of the statements (50%) referred to mastery goals and two (50%) to self-efficacy.

Herein are examples of motivational statements in the planning phase:

> Oded (Metacognitive Group): *I want to solve this problem* (mastery goals). *I can handle it by myself* (self-efficacy).
> Michal (Control Group): *I think I can solve it* (self-efficacy).

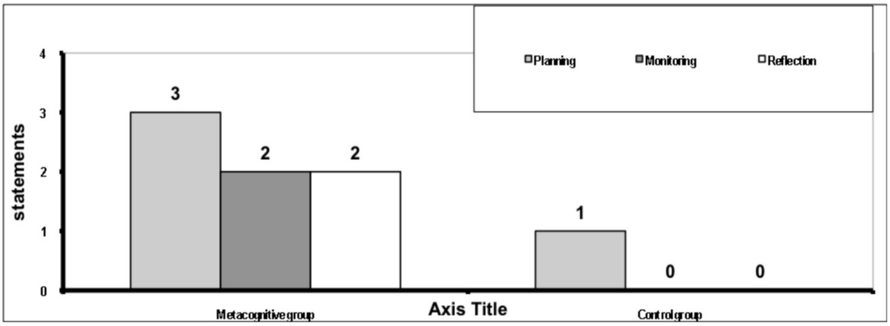

Figure 6. Frequency of motivational statements as a function of SRL phases in the Metacognitive and Control group

Monitoring phase

In this phase, motivational statements were found only in the Metacognitive Group. However, as in the planning phase, the use of these statements was very limited – only two statements (both from the same student) and both referring to mastery goals (see Table 1).

The motivational statements in the monitoring phase were as follows:

> Nathanael (Metacognitive Group): *I have to solve the problem* (mastery goals).
>
> Ore (Metacognitive Group): *It seems easy at first but then it's not... I want to solve it* (mastery goals).

Reflection phase

Motivational statements at this phase were found only in the Metacognitive Group (two students; Table 1). A total of two statements were found, one referred to self-efficacy and the other to mastery goals.

> Meni (Metacognitive Group): *I wanted to solve the problem because it was challenging but also interesting* (mastery goals).
>
> Oded (Metacognitive Group): *It was hard but I solved it* (self-efficacy).

Judgmental statements in each of the three SRL phases

As presented in Figure 7 the participants of both groups used judgmental statements. Furthermore, they used these statements in all SRL phases. However, the frequency of the statements differed between the two groups and the students of the Metacognitive Group used judgmental statements more frequently (n = 46, by all of students) than the students in the Control Group (n = 28, by 8 students). An interesting finding presented in Figure 7 indicated that most of the judgments in both groups were observed in the monitoring phase. This suggests that the students evaluated their solution path while judging their learning.

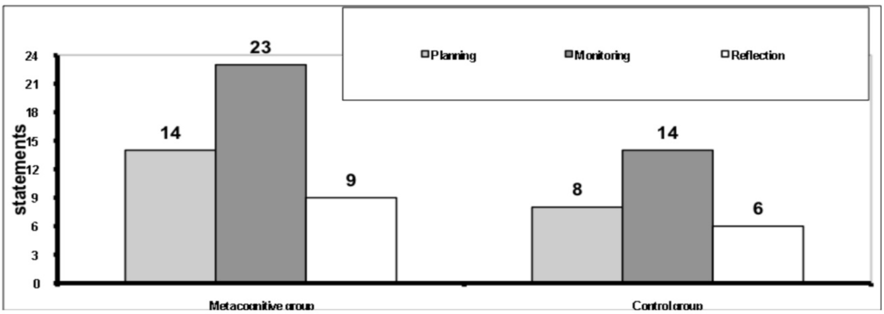

Figure 7. Frequency of judgmental statements as a function of SRL phase in the Metacognitive and Control group.

Planning phase

Analysis of the judgmental statements suggests that the students relied in their judgments on the data of the problem, their prior experience in solving problems and their mathematical knowledge. While in the Metacognitive Group twenty-five judgmental statements (76%) were found, in the Control Group only eight judgmental statements (24%) were found. Twelve of the students in the Metacognitive Group used judgmental statements (each student used approximately two statements), compared to seven students in the Control Group (see Table 1). All the judgmental statements at this phase referred to the easiness or difficulty of the learning task (EOL) and were found in both groups.

Herein are examples of judgmental statements in the planning phase:

> Koral (Metacognitive Group): *I read it once and the problem is difficult. I think I will solve it but I do not know how yet* (EOL)
> Matan (Control Group): *I am not sure I will solve it... there is a lot of data and it is complicated* (EOL).

Monitoring phase

The judgmental statements found at this phase were judgments of learning (JOL) and feeling of knowing (FOK). A total of 42 statements were found, of which 23 (55%) were in the Metacognitive Group and 19 (45%) in the Control Group. Twelve of the students in the Metacognitive Group used these types of sentences compared to ten students in the Control Group (see Table 1). It is important to note that JOLs were the most frequent judgments in this phase (35 statements, 9%) compared to FOK two statements (5%). This is probably due to the complexity of the non-routine problem. As students had not solved problems of this kind in the past and struggled to do so, they expressed their feelings and their concern regarding whether the path or the solution could be achieved. The judgments in this phase, as in the planning phase, were found in both groups but in different frequencies. The students relied in their judgments on the solution thus far, including their mathematical knowledge based on prior experience. They also expressed their doubts regarding the solution. Furthermore, it appeared that their judgments aided their decision-making process.

Herein are examples of judgmental statements made in the monitoring phase in both groups:

> Meni (Metacognitive Group): *The answer 6 is correct according to the data, but it doesn't make sense that a classroom will have 6 students* (JOL).
> Tomer (Metacognitive Group): *If I multiply, 6 by 5 I get 30, but 30 could be divided exactly and I need one child to be left alone... I do not*

understand .... (JOL).

Ronen (Metacognitive Group): *I don't think I've seen this sort of problems before* (FOK).

Leon (Control Group): *I thought 31 fits but no. I must have gone the wrong way* (JOL).

Following a close examination of the JOL judgmental statements in the monitoring phase, it seems that the Metacognitive Group was more aware of their own understanding of the task solution and clear regarding the strategy adaptation. However, no difference was found in the quantity of the JOL statements between both Metacognitive and Control groups.

Reflection phase

The judgmental statements at this phase referred to confidence (CJ) and were also found among both study groups. Nine (60%) of the statements were found in the Metacognitive Group and six (40%) of the statements in the Control Group. Eight of the students in the Metacognitive Group used these types of sentences compared to five students in the Control Group (see Table 1).

Herein are examples of judgmental statements in the reflection phase:

Tomer (Metacognitive Group): *I think I did well because I checked the answer* (CJ).

Shelly (Control Group): *I succeeded in the solution* (CJ).

Differences were found between the Metacognitive and Control groups in both the quantity and quality of the judgmental statements. The students in both groups referred to their satisfaction from the path of the solution they had followed while naming the challenge in solving the problem. However, the students in the Metacognitive Group better verbalized their feelings and presented a greater number of statements compared to the Control Group.

In conclusion, the findings indicate that, as with metacognitive statements, there were differences between the groups regarding the metacognitive judgments in the different SRL phases favoring the Metacognitive Group.

DISCUSSION

The present study aimed to highlight the role of SRL processes in the three phases (i.e., planning, monitoring and reflection) while solving a non-routine mathematical problem in a thinking aloud process. The question of the present study was whether it is possible for young students to benefit from "IMPROVE Guidance for

Metacognitive Judgments" regarding their ability to solve complex problems beyond what is learned during the intervention program, as was found in previous studies (Kramarski, Weisse, & Kololshi-Minsker, 2010; Kramarski et al., 2013; Marcou & Philippou, 2005; Zimmerman, 2008).

The data was collected with thinking aloud protocols in fourth-grade students. This is a methodological asset of the present study because it indicated that thinking aloud data can reveal spontaneous metacognitive, motivational and judgmental processes during problem solving.

Furthermore, it was found that students exposed to "Guidance for Metacognitive Judgments" embedded with the IMPROVE self-questions outperformed students in the Control Group and used more verbalizations indicative of metacognitive, motivational and judgmental statements than the students who followed the ordinary teaching methodology. This section discusses the contribution of the metacognitive guidance to problem solving and SRL components (metacognition with metacognitive judgments and motivation).

Mathematical non-routine problem solving and SRL-related statements

Professional literature suggests that providing learners, young and adult alike, with metacognitive tools, assists them in improving their achievement in the field of mathematics (Dignath et al., 2008; Kramarski et al., 2010; Kramarski et al., 2013; Tzohar-Rozen & Kramarski, 2013; Mevarech & Kramarski, 2014). The task chosen for the purposes of the study was a non-routine problem. Non-routine problems are, by definition, problems not studied before and require learners to demonstrate insight during the solution and draw conclusions (TIMSS, 2011). It seems that the making of disciplinary strategies explicit with IMPROVE self-questions (What, When, Why, How) can help students to think about the steps they need to take in their work, and help their thinking become explicit (Kramarski & Mevarech, 2003).

The advantage of the "Metacognitive Guidance" in an unprompted problem solving situation lies in the IMPROVE goal-driven processes that help link specific processes *(Comprehension* and *Strategy* questions) with generic processes *(Connection* questions) in repeated *Reflection* cycles for looking back and forward. These repeated cycles enabled students to be aware of both metacognitive aspects (knowledge, control) and help activate the two aspects in a complex task. These conclusions are in line with previous studies indicating the *power* of cyclical metacognitive models in intervention studies for students' gains (e.g., Kramarski et al., 2013; Zimmerman, 2008).

Metacognitive statements

Our findings indicated that the quantity and quality of the metacognitive statements were higher in all SRL phases (planning, monitoring, and reflection) in the Metacognitive Group compared to the Control Group. Another interesting finding was that the students in the Metacognitive Group used metacognitive statements (knowledge of cognition and control) in all SRL phases, as opposed to students in the Control Group who did not use metacognitive statements during the reflection phase. Additionally, *all* students in the Metacognitive Group used metacognitive statements compared to a small number of students in the Control Group. Furthermore, most of the metacognitive statements in the Metacognitive Group were found in the planning phase.

These findings regarding young students differ from previous findings in professional literature indicating a tendency to use the monitoring phase in mathematical problem solving. Specifically, in a study examining SRL among teachers of mathematics, Spruce and Bol (2015) found that the use of metacognition was common in the monitoring phase. Kramarski and Friedman (2014) also found that ninth-grade students demonstrated metacognitive talk relating to all three phases while solving word problems, with higher incidence in the monitoring phase. It is plausible that the non-routine problem used here demanded planning that led to a high frequency of metacognitive statements in the planning phase in both groups. In conclusion, our findings support our assumption and add to previous evidence from studies showing that metacognitive support of metacognitive self-questions assists learners in improving their knowledge of cognition and control in all SRL phases.

Motivational statements

The findings indicated that students of both groups did not make excessive use of motivational statements, although students in the Metacognitive Group used motivational statements more than those in the Control Group. Only one student in the Control Group used motivational statements as opposed to five students in the Metacognitive Group. The students in the Control Group used the motivational statement in the planning phase of the solution unlike the students in the Metacognitive Group who used these statements in all three phases.

These findings are supported by the study of Tzohar-Rozen and Kramarski (2013). In thir study no differences were found between the Metacognitive and Control Groups in motivation and self-efficacy among fifth-grade students who had followed a metacognitive intervention program while solving word problems in mathematics. It is important to note that these findings were based on questionnaires, which, as noted above, capture the learner's trait characteristics rather than real time motivation behavior during the solution of a specific problem. Thus, it can be concluded that metacognition by itself does not necessarily improve learners' motivation, as observed during the solution of *one* challenging non-routine problem.

Metacognitive judgments

Our findings indicated that students used judgment statements in all SRL phases (planning, monitoring, and reflection) in both groups, while the frequency of judgments in the Metacognitive Group was higher than in the Control Group.

Another interesting aspect is the differences between judgments in the various SRL phases such as planning, monitoring, and reflection. *In the planning phase,* judgments regarded the ease or difficulty of the learning task (EOL). The students relied on the data of the problem, their prior experience in solving similar problems and their mathematical knowledge to form their EOL judgment. At this phase, the Metacognitive Group made more EOL judgments compared to the Control Group. These findings are in line with previous studies (Cao & Nietfeld, 2005) indicating that judgments training, integrated in the content field, led to improvement in the accuracy of EOL judgments. However, the study by Cao and Nietfeld (2005) was conducted on college students, whereas the findings of the present study regarded young, fourth-grade students.

In the monitoring phase, judgment statements of JOL and FOK type were observed. The Metacognitive Group, as in the planning phase, used judgment statements more often than the Control Group. However, the evidence on JOL and FOK training is not conclusive and it seems that explicit training of such judgments does not always lead to the improvement of their accuracy (Hicks & Marsh, 2002; Logan et al., 2012; Townsend & Heit, 2011). It is important to note that in most studies, the training of JOLs and FOK judgments was not integrated in the teaching of a content field. The findings of the present study indicate that training for judgments integrated in the mathematics field, while cultivating the metacognitive component, increased the JOL and FOK judgments among young fourth-grade students.

In the reflection phase, confidence judgments (CJ) were used, and the students in the Metacognitive Group used such statements more than those in the Control Group. The participants of both groups referred to their satisfaction with the solution path while mentioning the challenge that accompanied it. These findings are in line with previous studies, which found that training for confidence judgments while teaching a content field contributes to its improvement (Huff & Nietfeld, 2009; Roderer & Roebers, 2010). Notably, the researchers found that following an intervention program integrating training for judgments, young students improved their abilities to assess the correctness of their performance after the solution (Roderer & Roebers, 2010). Thus, the findings of our study support the findings of previous studies and demonstrate an improvement in Confidence Judgments.

The differences in the judgmental component which favored the metacognitive group can be explained due to the explicit practice with the four rulers. In prior research, students exposed to concrete modeling are more willing to spend time and effort in adapting new approaches that help students succeed (Davis, 2003). Another explanation is that the "Judgmental rulers" in the metacognitive group exposed students to a data-driven process approach (Panadero, 2017) by using specific judgmental processes that appeared to enable students to experience local judgmental processes that may be more abstract for young learners. By underscoring local processes, the specific Judgmental rulers appeared to improve implicit mental models over practice – which helped students transfer this knowledge to a new context. Mental models are representations of situations and interrelations based on prior knowledge that can be easily applied to tasks in a new context (Hattie & Yates, 2014).

Contribution, limitations of the study and suggestions for future research

The novelty of the present study was the development of a unique intervention program that focused on "Guidance for Metacognitive Judgments" embedded in the IMPROVE metacognitive self-questions model oriented to SRL phases, while cultivating and developing metacognitive judgments, such as EOL, JOL, FOK and CJ. The outcomes of the intervention program were examined during the thinking-aloud solution of a non-routine mathematical problem. This is important because the study revealed spontaneous use of metacognitive judgments. A further contribution of the present study was the application of the intervention in a sample of young students with *all* judgmental types oriented to SRL phases in the mathematical domain, unlike most studies dealing with metacognitive judgments focusing on university students.

The findings of this study demonstrated the advantages of the intervention program, allowing learners to develop the metacognitive component and improve their ability in the content domain. This conclusion also applies in the field of metacognitive judgments and supports the recommendations of various researches, for instance, Ramdass and Zimmerman (2008), who opted to consider the student's ability in judgment during mathematical teaching. These researchers claim that solving problems requires use of metacognitive strategies along with judgment, which integrates decision making into the solution.

Still, the findings of the present study are limited due to the small number of participants and the use of qualitative analysis without any reference to quantitative data that could indicate possible improvement in the accuracy of judgments. Replication studies in other domains with a larger number of students or different types of students, e.g., students at risk of failure in learning mathematics or students with learning disabilities, are needed. In addition, it is recommended to examine the possibility of exploring temporal sequences of regulatory phases and not only frequencies to understand the tendency (Azevedo, 2014). To increase the reliability of the findings, video data should also be used. Further, it is important to check the impact of the intervention program in a follow up, not only after the end of the intervention. For instance, three months from the intervention.

In conclusion, it is important that future studies integrate different methodological tools directed to process data, such as thinking aloud, in the study of SRL. Focusing on data collected during the SRL process can lead to advancement of theory, methods and analytical techniques and, as result, to improvement of educational practice (Azevedo, 2014).

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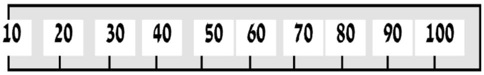
APPENDIX

Metacognitive guidance with "Judgmental rulers" directed to SRL phases

Judgment in the planning phase of the solution

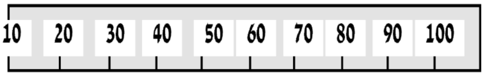
Ease of Learning – EOL: Judging the easiness or difficulty of the learning task *before* the solution.



Please read the problem. Before solving it, indicate the degree of your ability to succeed at solving the problem.

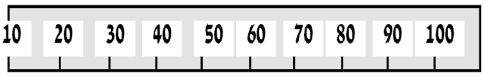
Judgments in the monitoring phase of the solution

1. **Judgment of learning – JOL:** Monitoring one's learning and understanding *during* the problem solution.



Indicate on the ruler the degree of your understanding of the problem.

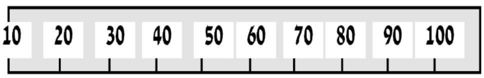
2. **Feeling of Knowing – FOK:** Monitoring one's feeling of knowledge of a certain topic/task *during* the solution.



Indicate on the ruler the degree of your knowledge of solving the problem.

Judgment in the reflection phase of the solution

Confidence Judgment – CJ: Assessing the correctness of one's performance *after* the solution.



Indicate on the ruler the degree of your success of solving the problem.